PRACTICAL SPARSE ARRAY DESIGN FOR SMALL SONARS

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Abstract: In digital sonar beamforming, the effective length of the array is the principle factor in determining the resolution of the system. Linear arrays enable 2D imaging and planar arrays, 3D imaging. The resolution of any array is proportional to array length and so for periodically spaced arrays resolution varies proportionally to cost for linear arrays and as the square of cost in planar arrays.

Sparse and random array design involves spacing array elements at irregular intervals and so can be of any length and hence any resolution, independent of cost. Additionally, since the element spacing is not periodic then grating lobes do not arise in the array beampattern. Such arrays do, however, have an unpredictable sidelobe structure and computational search techniques are often used to establish desirable array geometries.

This paper describes a method of array design which establishes array geometries by producing a highly constrained subset of solutions. This is seen as a practical alternative to computational search techniques which require careful design and development. Simulations of array geometries using this constrained technique are presented and compared to the standard equally-spaced configurations. Presently work is restricted to linear arrays.

Keywords: beamforming, arrays, sparse, random, non-periodic, sidelobe, sonar

1. INTRODUCTION

Digital sonar beamforming has traditionally utilised arrays of periodically spaced elements. The signals received by such arrays can be delayed and summed to produce directional gain. This directional gain can be steered across a scanning region and processed
to construct an image of the insonified area. A geometrically unambiguous 2D image can be constructed with a linear array but an unambiguous 3D image requires a planar array.

### 1.1 PERIODIC ARRAYS

If a received acoustic echo is assumed to be a complex sinusoidal plane wave incident upon a receiving array of elements of arbitrary geometry, then the signal intensity received at each element in the array can be described by equation 1 [1].

\[ x_i(n) = e^{j \frac{2\pi fn}{f_s} + kr \cdot u} \]

where:
- \( x \) = signal at \( i \)th element
- \( f \) = frequency of sinusoid (Hz)
- \( f_s \) = sampling frequency (Hz)
- \( n \) = temporal sample
- \( c \) = speed of sound in medium (m/s)
- \( i \) = element
- \( k = \frac{2\pi f}{c} \) = spatial frequency
- \( \Phi \) = azimuth (radians)
- \( \theta \) = elevation (radians)
- \( M \) = number of elements in array

The beampattern of the array results from the summation of the signals received by each element and is a function of scanning angle (\( u \)). Simplifying this for the case of a linear array and considering only 2D imaging, we can express the beampattern by:

\[ b(u) = \sum_{i=1}^{M} e^{jkr \sin \Phi} = \sum_{i=1}^{M} \cos(kr \sin \Phi) + j \sin(kr \sin \Phi) \]

All linear arrays with the same periodic spacing have similar beampatterns with recognised sidelobe structure and main lobe width regardless of element number. The main lobe width and hence angular resolution of an array depends principally on its length and so for periodically spaced arrays the resolution is dependent on the number of elements in the array. If the spacing of the elements in a periodic array is greater than half the wavelength of the acoustic pulse, spatial aliasing occurs resulting in grating lobes in the array beam pattern [2].

High resolution 2D imaging therefore requires long linear arrays and since element number is a key component in the cost of an array, resolution is proportional to cost. In 3D imaging, however, the resolution depends on the length of the array in both directions, so the cost increases as the square of the resolution [3]. This makes high resolution imaging in 2D and particularly 3D expensive and the spacing is often set to wavelength and the aliasing effects tolerated to offset the cost.

Since the resolution depends on array length, it is theoretically possible to have an array with few elements and very high resolution if the array elements are spaced non-periodically.
1.2 NON-PERIODIC ARRAYS

The sidelobe structure of non-periodic arrays is entirely unpredictable. Arrays of the same length and element number but differing geometries can have a vastly differing sidelobe structures. The removal of this periodicity, however, eliminates the presence of grating lobes in the array beampattern allowing for potentially alias-free high resolution imaging at a lower cost than standard periodic designs. [3]

There is no method by which a non-periodic array can be designed to produce a specific beampattern and traditionally such arrays have been designed on an ad hoc basis. Common approaches to the design of random arrays include search techniques such as genetic algorithms, simulated annealing and linear programming which search for optimal positions and weightings. Genetic algorithms were compared with simulated annealing in [4] and used in [5] to optimise array geometry and in [6] to optimise the array weighting. Linear programming was applied to optimise firstly the array weighting [7] and secondly both the array weighting and geometry. [8]

In this paper we propose an alternative to typical search techniques by attempting to constrain the number of possible element positions. This approach is based on the results of experiments with small arrays which revealed a relationship between element positions and optimal sidelobe levels.

2. EXPERIMENTS WITH SMALL ARRAYS

In an attempt to constrain the scale of the search for near optimal array geometries, it is appropriate to begin by investigating the effect of array geometry for small arrays. The optimal geometry and peak sidelobe level can only be established by searching all possible element configurations. Such a search (at an appropriate resolution) of linear 3 and 4 element arrays was conducted and the results analysed to see if any relationship could be established. Such a relationship may then be extrapolated to arrays of more elements. Results unveiled optimal array geometries which appear somewhat arbitrary.

A more intuitive approach, is to breakdown the equation describing an arbitrary array into real and imaginary parts (see equation 2) allowing the positions of the elements to be modelled by sinusoidal waves. If the equation is considered as a function of angle (u) then the frequency of a sinusoid is determined by element position, kr, and the phase by element position and beamsteering direction, -krui.

When this approach was used to analyse the results it was noted that in every case the optimal array geometry occurred when there was interference between the maxima and minima of the sinusoids used to model the element positions. Working back from the model we can establish what this means in terms of array geometry. Maxima or minima of the real and imaginary components of the beam pattern will occur whenever:

\[
\cos(kr_i (\sin \Phi - \sin \Phi_0)) = \pm 1 \\
k_r (\sin \Phi - \sin \Phi_0) = n_i \pi \\
r_{si} = \frac{n_i}{2(\sin \Phi - \sin \Phi_0)} \\
n_i = \pm 0, 1, 2, 3, \ldots, \infty
\]

\[
\sin(kr_i (\sin \Phi - \sin \Phi_0)) = \pm 1 \\
k_r (\sin \Phi - \sin \Phi_0) = n_i \pi \\
r_{si} = \frac{n_i}{2(\sin \Phi - \sin \Phi_0)} \\
n_i = \pm 0, 1, 2, 3, \ldots, \infty
\]
The 3 and 4 element experiments indicate that the interference of maxima and minima occur at the same scanning angle hence element positions which give rise to interfering peaks and troughs can be calculated as follows:

For any element, $r_i$, a peak or trough occurs at: 
\[ \sin \Phi = \frac{n_i}{2r_i} \lambda + \sin \Phi_0 \]

For the known element at the end of the array:
\[ \sin \Phi = \frac{n_{\text{max}}}{2r_{\text{max}}} \lambda + \sin \Phi_0 \]  
\[ (4) \]

The positions of the remaining elements can be calculated by:
\[ r_i = r_{\text{max}} \frac{n_i}{n_{\text{max}}} \]  
\[ (5) \]

Any element position multiplied by an integer gives a harmonic of that position (a spatial harmonic). Hence we can say that the optimal array geometries contain elements whose positions are harmonically related and allows us to introduce a new approach which we have called harmonic array design (HAD).

3. HARMONIC ARRAY DESIGN (HAD)

There are a countless number of element positions which are harmonically related. To reduce the number of possible positions, the two outermost elements in the array are predetermined, this not only fixes the array resolution but restricts the search to harmonically related element positions within the array length. For smaller arrays the number of solutions will be small enough for an exhaustive search of harmonic array geometries to be practical. This approach allows a good solution to be found without the need to develop and run a complex computational search algorithm.

The HAD technique was used to design small arrays and compared with the results from the exhaustive searches. In all cases the HAD technique produced a peak sidelobe level and array geometry that was near optimal. Fig.1 compares the peak sidelobe levels of 4 element arrays of various length for HAD arrays and those from the exhaustive search.

![Fig.1: Minimum peak sidelobe level exhaustive and HAD searches for 4 element experiments of differing array lengths](image)

Further experiments were conducted for 10 element sparse array geometries obtained using the HAD method. Table 1 lists the peak sidelobe and main beamwidth of such arrays.
along with that of periodically spaced arrays of equivalent length. The beampatterns of the sparse arrays are shown in Figs. 2 and 3.

These results demonstrate that the non-periodic array configuration matches the half wavelength spaced periodic array in terms of main beamwidth and has a slightly lower peak sidelobe. This demonstrates a slight performance enhancement for a non-periodic array in comparison with the half wavelength spaced periodic equivalent. They also show that a 10 element non-periodic array has a similar main beamwidth to a 10 element wavelength spaced periodic array or a 20 element half wavelength spaced periodic array. The periodic spaced arrays have peak sidelobes of around -13.3dB whilst the peak sidelobe of the non-periodic array is higher at -9.6dB. This increased peak sidelobe level of the non-periodic array has to be set against the cost saving of having only 10 elements instead of 20 (with a half wavelength spaced periodic array) and absence of grating lobes, which do occur in the beampattern of the wavelength spaced periodic array.

<table>
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<tr>
<th>elements</th>
<th>geometry</th>
<th>spacing</th>
<th>length ($\lambda$)</th>
<th>peak sidelobe (dB)</th>
<th>3dB beamwidth ($^\circ$)</th>
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<td>10</td>
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<td>half wavelength</td>
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<td>11.2</td>
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<td>9</td>
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<tr>
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<td>sparse</td>
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<td>-14.1</td>
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<tr>
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<td>sparse</td>
<td>n/a</td>
<td>9</td>
<td>-9.6</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 1: Experimental results of HAD and periodic arrays

4. CONCLUSIONS

The harmonic array design technique presented is a method of non-periodic array design which operates by constraining the number of solutions thus avoiding the need for a computational search algorithm. The HAD technique has been tested in small 3 and 4 element experiments and proven to achieve near optimal geometries in terms of peak sidelobe performance. The technique has also been used to create larger 10 element arrays and the performance of such arrays compared with their standard equivalents. It has been shown that the sparse array can match the angular resolution of periodic arrays and offers the advantage
of using fewer elements and avoiding ambiguity caused by grating lobes at the cost of an increased peak sidelobe level. Future work will consider extending experiments to planar arrays and researching the weighting functions in an effort to further reduce the peak sidelobe level of non periodic arrays.

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REFERENCES