USING SNELL’S LAW TO MEASURE SOUND SPEED DISPERSION

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Abstract: A technique has been developed to measure the speed of sound in marine sediments at discrete frequencies from 0.6 to 10 kHz by transmitting pulses from acoustic projectors within the water column and measuring the angle to which they are refracted on directional receivers buried in the seabed. The receivers are vectors sensors that measure three-axis acceleration and pressure allowing acoustic intensity, impedance, and arrival angle to be computed. A special jig was designed to bury four vector sensors into the seabed in two holes and control their orientation and depth of burial in the process. Following sensor burial, the jig was fully extracted to leave only the sensors and their associated cables in the seabed. The exact orientation of the sensors was confirmed by transmissions from three orthogonal directions, using two acoustic projectors also buried in the seabed by the jig and by an acoustic projector in the water column directly above the buried receivers. For the sound speed dispersion measurements, the seabed was ensonified at several grazing angles, concentrated above and below the anticipated critical angle. In this paper, the experimental approach and methods of data analysis are presented along with initial results from the SAX04 experiment in the Gulf of Mexico.

Keywords: wave speed dispersion, seabed acoustics, Snell’s Law

1. INTRODUCTION

Sediment sound speed measurements during several recent experiments indicate that the speed of sound travelling through marine sediments depends on the frequency of ensonification when the seabed is principally composed of sand [1]. In all cases, the speed of sound was found to increase with increasing frequency. This dispersion behaviour has implications for any application or research that involves the ratio between water and sediment sound speeds—the index of refraction. For example, it defines the critical angle,
important quantity in the detection of buried mines, and more generally in the science of underwater acoustics as it is used in modeling bottom loss, scattering, and propagation. The dispersive behaviour observed during SAX99 is generally consistent with Biot theory using model parameters based on geophysical measurements made at the site [1]. In fact, sound speed and attenuation measurements are being used as a fundamental metric to evaluate the predictions from different models for sound propagation in marine sediments. Measurements from 1 to 10 kHz are particularly important as the most significant differences in model behaviours are manifest in this band.

Below 10 kHz, the acoustic wavelengths under consideration effectively require that the sound speed measurements be made in situ. As a result there are very few measurements and they tend to have large uncertainties. This paper describes a technique developed by DRDC Atlantic and the Applied Research Laboratory at the Pennsylvania State University to measure sediment sound speed from approximately 0.6 to 10 kHz. It is one of five complementary approaches that are described elsewhere in these proceedings [2] along with the characteristics of the acoustic sources, receivers, and data acquisition systems that were employed. This paper begins with a description of the experimental concept, followed by the technique used to bury directional sensors in the seabed. Then, it explains how the data are analyzed to determine the arrival angle of acoustic transmissions, and ends with some preliminary results from the SAX04 experiment in the Gulf of Mexico.

![Fig. 1: Experimental geometry to measure sediment sound speed using Snell’s Law. A three-point mooring is used to adjust the grazing angle of sound from a projector in the water column. Buried vector sensors measure the angle of arrival of sound refracted into the seabed. Two buried sources are used to determine the orientation of the vector sensors. The receivers in the water column are used to determine the location of the water column source.](image-url)
2. EXPERIMENTAL CONCEPT

The technique is based on an application of Snell’s Law. The angle of refraction of sound into the seabed is a function of the sound speed ratio between the water and the seabed and the grazing angle of incidence (Fig. 1). As one approaches the critical angle for the seabed, 

\[ \theta_c = \arccos\left(\frac{c_w}{c_s}\right) \]

where \( c_w \) and \( c_s \) are the sound speed in water and sediment respectively, the frequency-dependent changes in angle of refraction are anticipated to become more pronounced. The grazing angle of incidence, of the Snell’s Law path, was varied using a three-point mooring system to change the position of an acoustic projector in the water column [2]. If the source location, receiver location, and water sound speed are known, then a measurement of the angle of refraction into the seabed may be used to estimate the sediment sound speed.

The arrival angle in the seabed was measured using buried vector sensors manufactured by Wilcoxon, model TV-001, each approximately 4 cm in diameter and 7 cm in length. The TV-001 sensor measures pressure and tri-axial acceleration thereby allowing acoustic intensity, impedance, and arrival angle to be computed (Section 4). The nominal sensitivity of the pressure sensor is -173 dB re 1\( \mu \)Pa/V and 1.1 V/g for the accelerometers. The sensitivities and beam-patterns of all components were calibrated at the discrete frequencies used in the experiments. Below 6 kHz, the accelerometer response is very close to that of an ideal dipole and that response is assumed in this initial examination of results. The frequency band over which this assumption is valid is discussed in Section 5.

3. SENSOR BURIAL

A critical part of this experimental approach is the ability to bury the vector sensors at known depths, with known orientations, and with known horizontal offsets relative to the acoustic sources. A burial jig (Fig. 2) was purpose built to bury four vector sensors and two spherical acoustic projectors. The burial jig consists of a metal framework, four casing tubes with PVC guide sleeves, mechanical aids to insert the tubes into the seabed, and tools to insert the sensors. The metal framework was assembled on the jetty before being loaded onto the research vessel to ensure that it was square and rigid. The distances between casing tubes were carefully measured (Fig. 2). The jig was placed on the seabed at the intersection point of two nylon ground lines (A-B and C-D) that ran between sand screws used to survey the site (Fig. 1). The sand screws were also used as the tether points for the three-point mooring, A-B-C, that held an acoustic projector in water (Fig. 1). As a result of this deployment approach, the ‘+y’ axis of the vector sensors is closely aligned with a projector that is constrained to move in the same range-depth plane (Fig. 1).

The burial jig was operated on the seabed by divers using the following sequence. First, it was leveled by adjusting scaffolding jack-screws in each corner and checking spirit levels fixed to the frame. Second, the four casing tubes were inserted into the seabed. To aid the insertion, the tubes had sharp teeth to cut through shells. The flanges on the casing tubes were designed to hold a pneumatic vibrator, but it was not required. A dredge tool extracted sediment from inside the casing tubes. Third, insertion tools were inserted into the casing tubes. Two tools, inserted into holes B and C, each held a pair of vector sensors between three tines for insertion at depths of 0.5 and 1.0 m. The tines were kept under tension by a loop of twine that ran to the top of the tool (eventually to be cut by the divers to release the sensors). Two tools, inserted into holes A and D, each held an ITC-1032 projector (6.9 cm in
diameter) at the end of a long rod for insertion at a depth of 1 m. The tools were fixed to the framework and keyed to prevent rotation. Fourth, the casing tubes were carefully retracted allowing sediment to cave in around the sensors and tools. Fifth, the sensors were released from the tools and then the tools were carefully retracted. Sixth, protective covers were put over the teeth on the casing tubes and the electrical leads pulled through the tubes. Lastly, the burial jig was removed leaving only the low-profile cabling for each sensor, fed vertically to the sediment interface, and run to the pressure vessel. During the SAX04 experiment, the sensors were buried on 14 October, 2004. They were left to equilibrate in the sediment until the first experiments were conducted on 26 October, 2004.

Fig. 2: a) Schematic drawing of the burial jig used to insert sensors into the seabed. b) Underwater photograph of Hole B, located adjacent to the intersection point of the nylon ground lines. The casing tube is retracted and insertion tools are holding the sensors in the seabed. c) A TV-001 sensor held by three tines that are kept under tension by the loop of twine. The safety clamp (right) is removed before insertion into the seabed.

4. ANGLE OF ARRIVAL

The arrival angle in a given plane may be estimated in two manners. The first determines the orientation of the major axis of the elliptical particle motion as measured by a pair of acceleration signals [3]. The second measures the direction to which the acoustic intensity vector is pointing [4]. In the frequency domain, if \( \tilde{a}(z,\omega,T) \) and \( \tilde{a}(r,\omega,T) \) are the complex amplitudes of acceleration (finite Fourier transforms over the record length \( T \)), in the vertical and horizontal in-line directions (where \( r \) is a linear combination of \( x \) and \( y \)) respectively, then the orientation of the major axis of elliptical motion is

\[
\theta_E(z,r,\omega) = \text{Re}\left\{\tan^{-1}\left(\frac{\tilde{a}(z,\omega,T)}{\tilde{a}(r,\omega,T)}\right)\right\},
\]

where \( \omega \) is the radian frequency. The ratio of ellipse semi-minor and semi-major axes is
\[-\tanh \left[ \text{Im} \left\{ \tan^{-1} \left( \frac{\hat{a}(z, \omega, T)}{\hat{a}(r, \omega, T)} \right) \right\} \right]. \tag{2} \]

To apply this to real data, one simply calculates the complex FFT of the sensor time series channels and divides the \(z\)-component by the \(r\) component, one frequency bin at a time. This estimate might be improved by averaging over the frequency bandwidth of the pulse. This has been tested with synthetic data, and the technique is robust against the addition of noise. The formula applies equally well to spectra of displacement, velocity, or acceleration.

Fig. 3: (a) Pressure and acceleration time series for a 5 ms duration acoustic pulse with a centre frequency of 1200 Hz. (b) Power spectra of the time series with a diamond symbol at the centre frequency. (c) Angle of arrival determined using the two spectral techniques described in the text. (d) and (e) Particle acceleration ellipses in the \(x-y\) (map view) and \(z-y\) (East-West vs. depth) planes. Arrival angle estimates using the "ellipse tilt" and "intensity vector" techniques are superimposed (dashed and solid black lines).

Vector sensors also allow the measurement of the acoustic intensity vector, the direction of which gives the direction of acoustic power flow. The single-sided intensity spectrum, \(\hat{G}_I\), is determined from the single-sided cross spectrum, \(\hat{G}_{pa}\), between the pressure signal and an accelerometer output [4]:
\begin{equation}
\tilde{G}_I(r,\omega) = \tilde{G}_{pu} = \frac{j}{\omega} \tilde{G}_{pa} = \frac{j}{\omega} \cdot \frac{2}{T} \left[ p(r,\omega,T) \cdot \tilde{a}^*(r,\omega,T) \right],
\end{equation}

where $p$ is the finite Fourier transform of the acoustic pressure and the asterisk denotes a complex conjugate. Eq. 3 gives the component of the intensity spectrum in the $r$ direction.

The arrival angle is obtained from the ratio of the imaginary parts (the active components) of the $z$ and $r$ intensity spectra:

\begin{equation}
\theta_I(z,r,\omega) = \tan^{-1} \left\{ \frac{\text{Im}(\tilde{G}_I(z,\omega))}{\text{Im}(\tilde{G}_I(r,\omega))} \right\}.
\end{equation}

To demonstrate these techniques for determining the angle of arrival, Fig. 3 contains a sample of data obtained during the SAX04 experiments in the Gulf of Mexico, about 2 km off the coast of Fort Walton Beach, Florida [5]. The experimental site was centred at 086°38.706’W, 30°23.232’N, approximately 80 m astern of the R/V Seward Johnson. The water depth was approximately 18 m. The acoustic arrivals were transmitted from an SX-100 acoustic projector [2] at a range of 7.32 m due West and height of 4.66 m above the seabed, and received on a TV-001 sensor (V1 in Fig. 1) buried in hole B (Fig. 2) at a depth of 0.5 m. The source location was determined using a regularized inversion technique that uses time-of-flight measurements between the source and four receivers in the water column [6]. The power spectra reveal that the measurement has excellent signal-to-noise ratio. The arrival angle estimates using Eqs. 1 and 4 are in close agreement throughout the bandwidth of the pulse. The arrival angles are superimposed on the particle acceleration ellipses (Figs. 3d and 3e) and are consistent with the orientation of their major axes.

The $x$-$y$ map view reveals that this sensor is rotated clockwise by several degrees. The sensor deployment technique (Fig. 2c) was very good at constraining their vertical orientation, but more susceptible to rotation in the $x$-$y$ plane. A comprehensive evaluation of sensor rotation is being undertaken using the arrival angles measured from the two sources buried in the seabed (Fig. 2a) and one moored directly overhead in the water column (Fig. 1). Once complete, the analysis of vertical arrival angles, $\theta_I$ and $\theta_E$, will be conducted with a linear combination of $x$ and $y$ components that are in-line with the source. In this paper, the dominant horizontal component, $y$, has been used for the analysis and display of the vertical arrival angles. The $z$-$y$ view reveals the arrival angle of the acoustic pulse in that plane after having been refracted into the seabed. It is these angle of refraction measurements that can be combined with the water sound speed and the source and receiver locations to estimate sediment sound speeds using Snell’s Law.

5. RESULTS

The techniques to determine arrival angle have been applied to pulses received at different discrete frequencies from 0.6 to 10 kHz. The results for sensor V3, buried at a depth of 50 cm in Hole C (Fig. 2), are shown in Fig. 4. The upper panels are the arrival angle in the $z$-$y$ and $x$-$y$ planes as a function of frequency. The lower panels are the width ratio (semi-minor/semi-major) of the ellipse axes and indicate the eccentricity of the elliptical motion.

In general, the arrival angle in the $z$-$y$ plane decreases as a function of increasing frequency. This indicates that the arrivals are coming in at progressively shallower angles as one might expect following Snell’s Law if the sound speed were increasing with frequency. Also shown in Fig. 4 is a prediction of the expected angle of refraction calculated using the Effective Density Fluid Model (EDFM) with input parameters from the SAX99 measurements [1] and a bottom water sound speed of 1534 m/s. Although the trend is clear and consistent with the SAX99 result, the data exhibit considerable structure, including a major excursion at 4 kHz. Future work will include a consideration of several possible
explanations including interference from a buried layer or the interaction between the refracted and evanescent fields. With regards to the latter explanation, the authors acknowledge that the application of Snell’s Law assumes that the energy flux is perpendicular to wave fronts. This is not always the case and the limits of this assumption are being investigated.

Fig. 4: The upper panels are the arrival angle, z-y and x-y planes, as a function of frequency. Arrival angle is determined using the “ellipse tilt” and “intensity vector” techniques. Symbols are the results for individual pings, with lines (red and black) through their median value. The Effective Density Fluid Model (EDFM) [1] is superimposed (blue line). The lower panels are the ratio of semi-minor and semi-major ellipse axes as a function of frequency.

Fig. 5: Sound speed measurements based on the angle of refraction of arrivals in the z-y plane (diamonds using the “ellipse tilt” and triangles using the “intensity vector” techniques). The results at 4 and 10 kHz are off the scale of this plot. The Effective Density Fluid Model (EDFM) [1] is superimposed (blue line).

The arrival angle in the x-y plane is relatively stable until 6 kHz where it deviates by a large amount. The localization of the acoustic source, using receivers in the water column and a generalized inversion technique [6], confirms that the projector remained stationary in the three-point mooring during the measurements at this station. The behaviour at 6 kHz and
above is because the accelerometer beam-patterns may no longer be considered to be idealized dipoles. This is consistent with the results of the pre-trial calibrations of the accelerometer beam-patterns, and with initial results from projector stations specifically designed to evaluate sensor performance by transmitting along the x, y, and z axes.

Applying Snell’s Law, sediment sound speed may be calculated using the arrival angles in the sediment (Fig. 4), the source and receiver positions, and a water sound speed of 1534 m/s. The result, plotted in Fig. 5, is not definitive. If one were to exclude the measurements at 1.6, 4, and 10 kHz (the latter because of the beam-pattern issues), then there is some suggestion of a dispersion relationship. However, the results at 1.6 and 4 kHz merit an explanation. A more thorough analysis of the data is underway, including an examination of the arrival angles on all four buried vector sensors and stations with different grazing angles of ensonification. In addition, the effects of reflections from a buried layer, interaction between direct and evanescent fields, and the conditions when Snell’s Law is applicable are all being considered.

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REFERENCES


